Angle	$0°$	$30°\left(\frac{\pi}{6}\right)$	$45°\left(\frac{\pi}{4}\right)$	$60°\left(\frac{\pi}{3}\right)$	$90°\left(\frac{\pi}{2}\right)$	
$\sin\theta$	$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{4}}=\frac{1}{2}$	$\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}}=1$	
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	formula $\tan\theta=\frac{\sin\theta}{\cos\theta}$
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\operatorname{cosec}\theta=\frac{1}{\sin\theta}$
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	cosec और Sec

Some formula prove

(i) $\sin\theta \times \operatorname{cosec}\theta = 1$

Prove

$$L.H.S = \sin\theta \times \operatorname{cosec}\theta$$

$$= \frac{P}{h} \times \frac{h}{P} = 1 \quad R.H.S \text{ proved}$$

(ii) $\cos\theta \times \sec\theta = 1$

Proved

$$L.H.S = \cos\theta \times \sec\theta$$

$$= \frac{b}{h} \times \frac{h}{b} = 1 \quad R.H.S$$

Proved

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(iii) $\tan\theta \times \cot\theta = 1$

L.H.S $= \frac{P}{b} \times \frac{b}{P} = 1$ R.H.S proved

(iv) $\sin\theta \times \operatorname{cosec} = 1$

$\Rightarrow \sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\Rightarrow \operatorname{cosec}\theta = \frac{1}{\sin\theta}$

(v) $\cos\theta \times \sec\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta}$

$\Rightarrow \sec\theta = \frac{1}{\cos\theta}$

(vi) $\tan\theta \times \cot\theta = 1$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta}$

$\Rightarrow \cot\theta = \frac{1}{\tan\theta}$

(vii) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

(viii) $\cot = \frac{\cos\theta}{\sin\theta}$

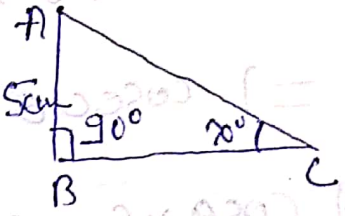
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* 0° से 90° की ओर जाने पर Sin θ, tan θ, Sec θ का मान बढ़ता है।
* 0° से 90° की ओर जाने पर cos θ, cot θ, cosec θ का मान घटता है।

— उदाहरण — 8.25

उदाहरण ⑥

Solve



△ ABC में,

$\angle B = 90°$

$\angle ACB = 30°$

$BC = ?$

$AC = ?$

$\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BC}$

$\therefore BC = 5\sqrt{3}$ cm Ans

$\sin 30° = \frac{AB}{AC}$

or, $\frac{1}{2} = \frac{5}{AC}$

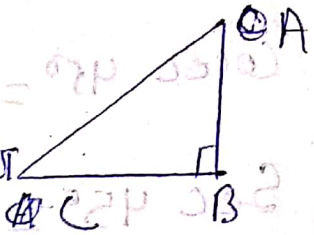
$\therefore AC = 10$ cm

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V.V.I

⑥ 45° का त्रिकोणमितिय अनुपात में मान
(Value of 45° trigonometry ratio)

माना कि ABC एक समद्विबाहु समकोण Δ है/ जिसमें $\angle B = 90°$ है/ ~~जिसमें~~ और $AB = CB$



$\because AB = BC$

$\angle A = \angle C$ (∵ बराबर भुजा के सम्मुख कोण बराबर होती है)

∵ Δ के तीनों कोणों का योग 180° होता है/

$\therefore \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 90° + \angle A = 180°$ (∵ $\angle A = \angle C$)

$\Rightarrow 2\angle A = 180° - 90°$

$\Rightarrow 2\angle A = 90°$

$\therefore \angle A = \frac{90°}{2} = 45°$

$\Rightarrow \angle A = 45°$

माना कि $AB = BC = x$

समकोण ΔABC में पाइथागोरस प्रमेय से,

$\Rightarrow AC^2 = AB^2 + BC^2$ $\Rightarrow AC = \sqrt{2x^2}$

$\Rightarrow AC^2 = x^2 + x^2$ $\Rightarrow AC = x\sqrt{2}$

$\Rightarrow AC^2 = 2x^2$

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$\sin 45^\circ = \sin C = \frac{AB}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos 45^\circ = \cos C = \frac{BC}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\tan 45^\circ = \tan C = \frac{AB}{BC} = \frac{x}{x} = 1$

$\operatorname{cosec} 45^\circ = \operatorname{cosec} C = \frac{AC}{AB} = \frac{x\sqrt{2}}{x} = \sqrt{2}$

$\sec 45^\circ = \sec C = \frac{AC}{ABC} = \frac{x\sqrt{2}}{x} = \sqrt{2}$

$\cot 45^\circ = \cot C = \frac{BC}{AB} = \frac{x}{x} = 1$

Q $\sin 45^\circ$ का मान ज्यामिति विधि से ज्ञात करें

Solveluction

माना कि ABC एक समद्विबाहु समकोण $\triangle$ है

जिसमें $\angle B = 90^\circ$ और $AB = BC$

$\therefore \angle A = \angle B$ (प्रमेय से)

$\therefore \triangle$ के तीनों कोणों का योग $= 180^\circ$

$\therefore \angle A + \angle B + \angle C = 180^\circ$ (प्रमेय से)

$\Rightarrow \angle A + 90^\circ + \angle A = 180^\circ$

$\Rightarrow 2\angle A = 180^\circ - 90^\circ$

$\Rightarrow \angle A = \frac{90^\circ}{2} = 45^\circ$

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माना कि $AB = BC = x$

समकोण $\triangle ABC$ में पाइथागोरस प्रमेय से,

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = x^2 + x^2$

$\Rightarrow AC = \sqrt{2x^2}$

$\Rightarrow AC = x\sqrt{2}$

$\sin 45^\circ = \frac{AB}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$

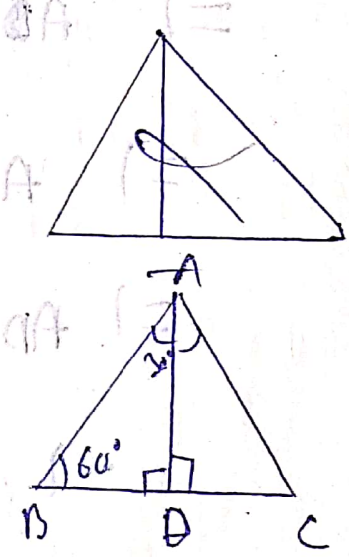
Q 30° का त्रिकोणमितिय अनुपात का मान
(Value of 30° trigonometric ratio)

माना कि ABC एक समबाहु $\triangle$ है।

∴ ABC एक समबाहु $\triangle$ है।

$\angle A = \angle B = \angle C = 60^\circ$

शीर्ष A से $AD \perp BC$ खींचा।



$\triangle ABD$ तथा $\triangle ADC$ में,

$\angle ADB = \angle ADC = 90^\circ$

$AD = AD$ (उभयनिष्ठ)

$AB = AC$ (समबाहु त्रिभुज के कर्ण हैं)

R.H.S से

$\triangle ABD \cong ACD$

$\therefore BD = CD$ ——— (i)

$\angle BAD = \angle CAD$ ——— (ii)

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∵ $\angle BAC$ या $\angle A = 60°$

∴ $\angle BAD$ या $\angle A = 30°$

माना कि $AB = BC = AC = x$

∴ $BC = x$

∴ $BD = CD = \frac{x}{2}$

पाइथागोरस प्रमेय से,

$AB^2 = AD^2 + BD^2 = x^2 + \frac{x^2}{4}$

$\Rightarrow AD^2 = \frac{-x^2 + 4x^2}{4}$

$\Rightarrow AD = \frac{3x^2}{4}$

$\Rightarrow AD = \sqrt{\frac{3x^2}{4}}$

$\Rightarrow AD = \frac{x\sqrt{3}}{2}$

$\sin 30° = \frac{BD}{AB} = \frac{\frac{x}{2}}{x} = \frac{x}{2x} = \frac{1}{2}$

$\cos 30° = \frac{AD}{AB} = \frac{\frac{x\sqrt{3}}{2}}{x} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{BD}{AD} = \frac{\frac{x}{2}}{\frac{x\sqrt{3}}{2}} = \frac{x}{2} \times \frac{2}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

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$$\text{Cosec } 30^\circ = \frac{AB}{BD} = \frac{x}{\frac{x}{2}} = \frac{2x}{x} = 2$$

$$\text{Sec } 30^\circ = \frac{AB}{AD} = \frac{x}{\frac{x\sqrt{3}}{2}} = \frac{2x}{x\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\cot 30^\circ = \frac{AD}{BD} = \frac{\frac{x\sqrt{3}}{2}}{\frac{x}{2x}} = \frac{x\sqrt{3}}{2} \times \frac{x}{x} = \sqrt{3}$$

Q यदि $\cos\theta + \sin\theta = 1$ तो सिद्ध करें कि $\cos\theta - \sin\theta = \pm 1$

Solve

माना कि $\cos\theta - \sin\theta = x$

दोनों तरफ वर्ग करने पर

$$(\cos\theta - \sin\theta)^2 = x^2$$

$$\Rightarrow \cos^2\theta - \sin^2\theta + 2(\cos\cdot\sin)\theta = x^2$$

$$\Rightarrow 1 - 2\cos\cdot\sin\theta = x^2 \quad \text{---------- (1)}$$

$\Rightarrow 1$ $\because \cos\theta + \sin\theta = 1$

दोनों तरफ वर्ग करने पर

$$\Rightarrow (\cos\theta + \sin\theta)^2 = 1^2$$

$$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cdot\cos\theta\cdot\sin\theta = 1$$

$$\Rightarrow 1 + 2\sin\theta\cdot\cos\theta = 1$$

$$\Rightarrow x - x + 2\sin\theta\cdot\cos\theta = 0$$

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$\Rightarrow 2\sin\theta \cdot \cos\theta = 0$ ——— (iii)

समीकरण (ii) और (i) में मान रखने पर

$1 - 0 = x^2$

$\Rightarrow 1 = x^2$

$\Rightarrow x = \pm\sqrt{1}$

$\Rightarrow x = \pm 1$

$\cos\theta - \sin\theta = 1$

Note — यदि कुछ विशिष्ट कोणों का त्रिकोणमितीय अनुपात का मान रखने पर से हर में अपरिमेय संख्या आये तो इसका परिमेयकरण करेंगे

$y =$ यदि हर में 1 पद है तो इस पद के अपरिमेय संख्या से अंश और हर को गुणनफल करेंगे तो हर का परिमेयकरण हो जायेंगे।

$$\frac{1}{2\sqrt{a}} = \frac{1}{2\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{2a}$$

Note — यदि हर में दो पद है तो इस पद का conjugate से हर और अंश को गुणा करेंगे तो हर का परिमेयकरण हो जायेंगे।

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Conjugate :- $a+b$ का Conjugate $a-b$ होगा और $a-b$ का Conjugate $a+b$ होता है।

Note :- (iii) :- यदि हर में $-$ आ जाय तो इस $-$ को हटाने के लिए -1 से अंश और हर को गुणा करेंगे। जैसे :- $\frac{2}{-5} \times \frac{-1}{-1} = \frac{-2}{5}$

Q उदाहरण (7)

Solve

$\triangle PQR$ में

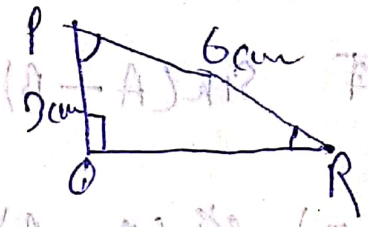
$\angle Q = 90°$

$PQ = 3cm$

$PR = 6cm$

$\angle QPR = ?$

$\angle PRQ = ?$



$$\cos P = \frac{b}{h} = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2}$$

$\Rightarrow \cos P = \frac{1}{2}$

$\Rightarrow \cos P = \cos 60°$

$\therefore \angle P = 60°$

अतः $QPR = 60°$

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फिर, $\sin R = \frac{P}{h} = \frac{PQ}{PR} = \frac{3}{6_2} = \frac{1}{2}$

$\therefore \sin R = \sin 30^\circ$

$\Rightarrow \angle R = 30^\circ$

$\therefore \angle PRQ = 30^\circ$

उदाहरण (8.)

Solvelution

दिया है

$\sin(A-B) = \frac{1}{2}$

$\cos(A+B) = \frac{1}{2}$

अब $\sin(A-B) = \frac{1}{2}$

$\Rightarrow \sin(A-B) = \sin 30^\circ$

$\Rightarrow A - B = 30^\circ$ ——— (i.)

फिर, $\cos(A+B) = \frac{1}{2}$

$\Rightarrow \cos(A+B) = \cos 60^\circ$

$\Rightarrow A + B = 60^\circ$ ——— (ii.)

समीकरण (i.) और (ii.) को जोड़ने पर

$A - B + A + B = 30^\circ + 60^\circ$

$\Rightarrow 2A = 90^\circ$

$\therefore A = \frac{90^\circ}{2} 45^\circ = \angle A = 45^\circ$

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समीकरण (i) में $\angle A$ का मान रखने पर

$A - B = 30°$

$\Rightarrow 45° - B = 30°$

$\Rightarrow -B = 30° - 45°$

$\Rightarrow +B = +15$

$\therefore \angle B = 15$

अतः $\angle A = 45°$, $\angle B = 15°$ Ans

प्रश्नावली — 8.2

1.(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{3+1}{4} = \frac{4}{4} = 1$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4} = 2$

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$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} = \frac{\sqrt{3} \times 2\sqrt{2} - 2\sqrt{6}}{(2\sqrt{2}+2\sqrt{6})(2\sqrt{2}-}$$

$$= \frac{2\sqrt{6} - 2\sqrt{18}}{(2\sqrt{2})^2 - (2\sqrt{6})^2} = \frac{2\sqrt{6} - 2\sqrt{18}}{8 - 24} = \frac{2\sqrt{6} - 2\sqrt{18}}{-16}$$

$$= \frac{2\sqrt{2\times3} - 2\sqrt{2\times3\times3}}{-16} = \frac{-2\sqrt{2\times3\times3} + 2\sqrt{6}}{-16} = \frac{-6\sqrt{2}+}{-16}$$

$$= \frac{+2(3\sqrt{2} - \sqrt{6})}{+16\ 8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

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(14)

$$= \frac{\frac{1+2-1}{2}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}} \times \frac{2\sqrt{3}}{4+\sqrt{3}+2\sqrt{3}} = \frac{\sqrt{3}+2\sqrt{3}-4}{4+\sqrt{3}+2\sqrt{3}}$$

$$= \frac{3\sqrt{3}-4}{4+3\sqrt{3}} \times \frac{4-3\sqrt{3}}{4-3\sqrt{3}} = \text{yeh step coong hui}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4} = \frac{(3\sqrt{3})^2 - 2\cdot 3\sqrt{3}\cdot 4 + (4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$= \frac{27 - 24\sqrt{3} + 16}{27-16} = \frac{43 - 24\sqrt{3}}{11} \text{ Ans}$$

(VI) $$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{79-12}{12} = \frac{67}{12} \text{ Ans}$$

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(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{3\sqrt{3}}{2\times 3}$

$\frac{\sqrt{3}}{2} = \sin 60^\circ$

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-(1)^2}{1+1^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0 = \sin$

(iii) $\sin 2A = 2\sin A$

$\because \sin 2A = 2\sin A$

$A = 0$

$L.H.S = \sin 2A = \sin \times 2 \times 0 = \sin 0$

$R.H.S = 2\sin A = 2 \times \sin 0 = 0$

$\sin 0 \longrightarrow$ (i) 0°

$L.H.S = R.H.S \longrightarrow$ (ii) $\tan 60^\circ$

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60^\circ$

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Solve Question

$\because \tan(A+B) = \sqrt{3}$

$\therefore \tan(A+B) = \tan 60^\circ$

$\Rightarrow A+B = 60^\circ$ ——— (i)

फिर, $\because \tan(A-B) = \frac{1}{\sqrt{3}}$

$\therefore \tan(A-B) = \tan 30^\circ$

$\Rightarrow A-B = 30^\circ$ ——— (ii)

समीकरण (i) और समीकरण (ii) को जोड़ने पर

$(A+B)+(A-B) = 60^\circ + 30^\circ$

$\Rightarrow A+B+A-B = 90^\circ$

$\Rightarrow 2A = 90^\circ$

$\therefore A = \frac{90^\circ}{2} = 45^\circ$

समीकरण (i) में A का मान रखने पर

$\Rightarrow A+B = 60^\circ$

$\Rightarrow 45+B = 60^\circ$

$\Rightarrow B = 60^\circ - 45^\circ$

$\Rightarrow B = 15^\circ$

$A = 45^\circ$
$B = 15^\circ$ } Ans

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(4) (i) $\sin(A+B) = \sin A + \sin B$

माना कि $A = 60°$, $B = 30°$

L.H.S = $\sin(A+B)$

$= \sin(60° + 30°)$

$= \sin 90°$

$= 1$

R.H.S = $\sin A + \sin B$

$= \sin 60° + \sin 30°$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2}$

$\therefore$ L.H.S = R.H.S

$\therefore$ असत्य

(ii) Ans: — सत्य, क्योंकि θ वृद्धि करने से Sin का मान 1 तक बढ़ता है।

(iii) Ans: — असत्य, क्योंकि θ वृद्धि करने से Cos का मान बढ़ाने से Cos का मान 1 से 0 तक घटता है।

(iv) Ans: — असत्य, क्योंकि $\theta = 45°$ लेने से $\sin\theta = \cos\theta$ होता है अन्य मान के लिए $\sin\theta = \cos\theta$ नहीं होता है।

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(v) Ans:— सत्य, क्योंकि $\cot\theta$ = अपरिभाषित या परिभाषित नहीं है

(i) असत्य ⟶ (i) $\sin(A+B) = \sin A + \sin B$

माना कि $A = 60°$, $B = 30°$ है

$\sin(60°+30°) = \sin 60° + \sin 30°$

$\Rightarrow \sin 90° = \frac{\sqrt{3}}{2} + \frac{1}{2}$

$\Rightarrow 1 = \frac{\sqrt{3}+1}{2}$

(ii) सत्य ⟶ (ii)

Angle	30°	45°	60°	90°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

(iii) असत्य ⟶ (iii)

(iv) असत्य ⟶ (iv) $\sin\theta \neq \cos\theta$

(vi) सत्य ⟶ (v) $A = 0°$, तो $\cot A = \infty$ (infinity)

formula (i) $\sin(90° - \theta) = \cos\theta$

(ii) $\cos(90° - \theta) = \sin\theta$

(iii) $\tan(90° - \theta) = \cot\theta$

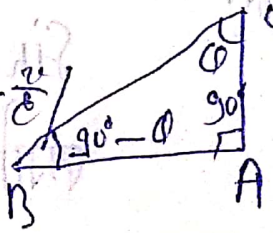
(iv) $\cot(90° - \theta) = \tan\theta$

(v) $\operatorname{cosec}(90° - \theta) = \sec\theta$

(vi) $\sec(90° - \theta) = \operatorname{cosec}\theta$

(i) proof:—

माना कि $\triangle ABC$ एक समकोण त्रिभुज है जिसमें $\angle A = 90°$, $\angle C = \theta$, $\angle B = 90° - \theta$



तो सिद्ध करना है कि $\sin(90° - \theta) = \cos\theta$

proof:— $\because \sin B = \frac{AC}{BC}$

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$\Rightarrow \sin(90^\circ - \theta) = \frac{AC}{BC}$ ——— (i)

फिर, $\cos C = \frac{AC}{BC}$

$\Rightarrow \cos\theta = \frac{AC}{BC}$ ——— (ii)

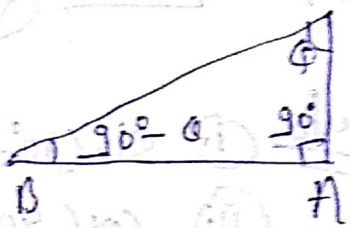
समीकरण (i) और (ii) से

$\sin(90^\circ - \theta) = \cos\theta$ Proved

Q (iii) सिद्ध करें कि $\cos(90^\circ - \theta) = \sin\theta$

माना कि ΔABC एक समकोण त्रिभुज है

जिसमें $\angle A = 90^\circ$, $\angle C = \theta$, $\angle B = (90^\circ - \theta)$



तो सिद्ध करना है कि $\cos(90^\circ - \theta) = \sin\theta$

proof :— $\because \cos B = \frac{AB}{BC}$

$\Rightarrow \cos(90^\circ - \theta) = \frac{AB}{BC}$ ——— (i)

फिर,

$\sin C = \frac{AB}{BC} \Rightarrow \sin\theta = \frac{AB}{BC}$ ——— (ii)

समीकरण (i) और (ii) से

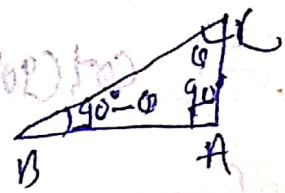
$\frac{\cos(90^\circ - \theta)}{\sin\theta} = \frac{\frac{AB}{BC}}{\frac{AB}{BC}} = \cos(90^\circ - \theta) = \sin\theta$

Proved

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(iii) सिद्ध करें कि $\tan(90° - \theta) = \cot\theta$ है।

माना कि ΔABC एक समकोण त्रिभुज है जिसमें $\angle A = 90°$, $\angle B = 90° - \theta$, $\angle C = \theta$ है।



तो सिद्ध करना है कि $\tan(90° - \theta) = \cot\theta$

$\because \tan B = \frac{AC}{AB}$

$\Rightarrow \tan(90° - \theta) = \frac{AC}{AB}$ ——(i)

फिर, $\cot C = \frac{AC}{AB}$

$\Rightarrow \cot\theta = \frac{AC}{AB}$ ——(ii)

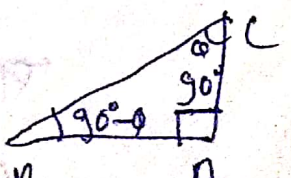
समी (i) और (ii) से,

$$\frac{\tan(90° - \theta)}{\cot\theta} = \frac{\frac{AC}{AB}}{\frac{AC}{AB}}$$

$\Rightarrow \tan(90° - \theta) = \cot\theta$ Proved

(iv) $\cot(90° - \theta) = \tan\theta$

माना कि ΔABC एक समकोण त्रिभुज है जिसमें $\angle A = 90°$, $\angle B = 90° - \theta$, $\angle C = \theta$



तो सिद्ध करना है कि ~~$\tan(90° - \theta) = \cot\theta$~~ $\cot(90° - \theta) = \tan\theta$

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$\because \cot B = \frac{AB}{AC}$

$\Rightarrow \cot(90° - \theta) = \frac{AB}{AC}$ ——— (i)

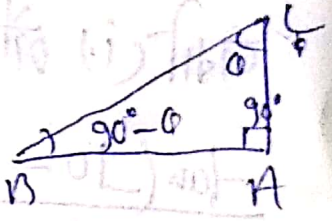
फिर, $\tan\theta = \frac{AB}{AC}$

$\Rightarrow \tan\theta = \frac{AB}{AC}$ ——— (ii)

समीकरण (i) और (ii) से,

$\cot(90° - \theta) = \tan\theta$ proved

(v) माना कि $\triangle ABC$ एक समकोण त्रिभुज है जिसमें $\angle A = 90°$, $\angle B = 90° - \theta$, $\angle C = \theta$ है।



तो सिद्ध करना है कि $\operatorname{cosec}(90° - \theta) = \sec\theta$

$\because \operatorname{cosec} B = \frac{BC}{AC}$

$\Rightarrow \operatorname{cosec}(90° - \theta) = \frac{BC}{AC}$

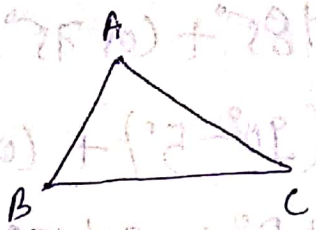
फिर $\because \sec C = \frac{BC}{AC}$

समी (i) को समी (ii) में भाग देने पर

$\operatorname{cosec}(90° - \theta) = \sec\theta$ Proved

ΔABC में

$A + B + C = 180^\circ$



Dividing both the sides by 2.

or, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{180}{2}^{90}$

or, $\frac{B}{2} + \frac{C}{2} = (90^\circ - \frac{A}{2})$

or, $\left(\frac{B+C}{2}\right) = (90^\circ - \frac{A}{2})$

Taking sine both the sides,

or, $\sin\left(\frac{B+C}{2}\right) = \sin(90^\circ - \frac{A}{2})$

or, $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$ proved

उदाहरण (9) $\frac{\tan 65^\circ}{\cot 25^\circ} = \frac{\tan 65^\circ}{\cot(90^\circ - 65^\circ)} = \frac{\tan 65^\circ}{\tan 65^\circ} = 1$

उदाहरण (10) $\sin 3A = \cos(A - 26^\circ)$

$\Rightarrow \cos(90^\circ - 3A) = \cos(A - 26^\circ)$

or, $90^\circ - 3A = A - 26^\circ$

or, $-3A - A = -26^\circ - 90^\circ$

or, $-4A = -116^\circ$

$\therefore A = \frac{116^{29}}{4}^\circ = 29^\circ$

$\therefore A = 29^\circ$

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उदाहरण (10) $\cot 85° + \cos 75°$

$= \cot(90° - 5°) + \cos(90° - 15°)$

$= \tan 5° + \cot 15°$

प्रश्नावली – 8.3

(7) (i) $\frac{\sin 18}{\cos 72°} = \frac{\sin 18°}{\cos(90° - 18°)} = \frac{\sin 18°}{\sin 18°} = 1$

(ii) $\frac{\tan 26°}{\cot 64°} = \frac{\tan 26°}{\cot(90° - 26°)} = \frac{\tan 26°}{\tan 26°} = 1$

(iii) $\cos 48° - \sin 42°$

$= \cos 48° - \sin(90° - 48°)$

$= \cos 48° - \cos 48°$

(iv) $\operatorname{cosec} 31° - \sec 59°$

$= \operatorname{cosec} 31° - \sec(90° - 31°) = \operatorname{cosec} 31° - \operatorname{cosec} 31°$

(v)

(2) Prove that

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

L.H.S $= \tan 48° \tan 23° \tan(90° - 48°) \tan(90° - 23°)$

$= \tan 48° \tan 30° \cot 48° \cot 23°$

$= 1 \cdot 1 = 1$ R.H.S proved

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(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

L.H.S $= \cos 38^\circ \operatorname{cosec} 52^\circ - \sin(90^\circ - 52^\circ) \cdot \sin(90^\circ - 38^\circ)$

$= \cos 38^\circ \operatorname{cosec} 52^\circ - \cos 38^\circ \cos 52^\circ$

$= 0$ R.H.S proved

(3) Solve

$\tan 2A = \cot(A - 18^\circ)$

$\Rightarrow \cot(90^\circ - 2A) = \cot(A - 18^\circ)$

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow -2A - A = -18^\circ - 90^\circ$

$\Rightarrow -3A = -108^\circ$

$\Rightarrow A = \frac{108}{3} = 36^\circ$

$\therefore A = 36^\circ$

(4) solveution

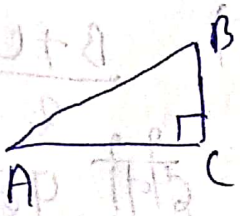
$\tan A = \cot B$

$\Rightarrow \cot(90^\circ - A) = \cot B$

$\Rightarrow 90^\circ - A = B$

$\Rightarrow 90^\circ = A + B$

$\Rightarrow A + B = 90^\circ$ proved



(5) solveludion

$\operatorname{Sec} 4A = \operatorname{Cosec}(A - 20^\circ)$

$\Rightarrow \operatorname{Cosec}(90^\circ - 4A) = \operatorname{Cosec}(A - 20^\circ)$

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$\Rightarrow 90^\circ - 4A = A - 20^\circ$

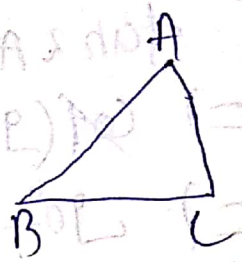
$\Rightarrow -4A - A = -20^\circ - 90^\circ$

$\Rightarrow +5A = +110$

$\Rightarrow A = \frac{110}{5} = 22$

$\therefore A = 22^\circ$

⑥ Solveution



ΔABC में,

$A + B + C = 180^\circ$ (प्रमेय से)

दोनों ओर 2 से भाग देने पर

$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{180^\circ}{2} 90^\circ$

$\Rightarrow \frac{B}{2} + \frac{C}{2} = 90^\circ - \frac{A}{2}$

$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$

दोनों पक्षों का sin लेने पर

$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$ proved

⑦ solveution

$\sin 67^\circ + \cos 75^\circ$

$= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$

$= \cos 23^\circ + \sin 15^\circ$

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