Some formula

(1) $\sin^2\theta + \cos^2\theta = 1$

(2) $\sec^2\theta - \tan^2\theta = 1$

(3) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Prove that (1) $\sin^2\theta + \cos^2\theta = 1$

L.H.S $= \sin^2\theta + \cos^2\theta$

$= \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{p^2 + b^2}{h^2} = \frac{h^2}{h^2} = 1$

R.H.S proved

Prove that (2) $\sec^2\theta - \tan^2\theta = 1$

L.H.S $= \left(\frac{h}{b}\right)^2 - \left(\frac{p}{b}\right)^2 = \frac{h^2}{b^2} - \frac{p^2}{b^2} = \frac{h^2 - p^2}{b^2} = \frac{b^2}{b^2} = 1$

R.H.S proved

Prove that (3) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

L.H.S $= \left(\frac{h}{p}\right)^2 - \left(\frac{b}{p}\right)^2 = \frac{h^2}{p^2} - \frac{b^2}{p^2} = \frac{h^2 - b^2}{p^2} = \frac{p^2}{p^2} = 1$

R.H.S proved

(1) $\sin^2\theta + \cos^2\theta = 1$

(i) $\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \sin\theta = \sqrt{1 - \cos^2\theta}$

(ii) $\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$

(2) $\sec^2\theta - \tan^2\theta = 1$

(i) $\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec\theta = \sqrt{1 + \tan^2\theta}$

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(ii) $\Rightarrow \tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$

(iii) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

(i) $\Rightarrow \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \operatorname{cosec}\theta = \sqrt{1 + \cot^2\theta}$

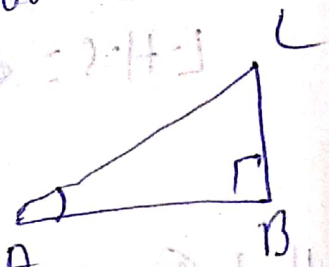
(ii) $\Rightarrow \cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\Rightarrow \cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}$

Some important formulae

(I) त्रिभुज के 2 कोण :—

(i) $\operatorname{cosec} A = \frac{1}{\sin A}$



(ii) $\sec A = \frac{1}{\cos A}$ Prove that (ii) $\operatorname{cosec} A = \frac{1}{\sin A}$

(iii) $\cot A = \frac{1}{\tan A}$

$\Rightarrow \operatorname{cosec} A \cdot \sin A = 1$

(iv) $\tan A = \frac{\sin A}{\cos A}$

$\Rightarrow \frac{AC}{BC} \times \frac{BC}{AC} = 1$

(v) $\cot A = \frac{\cos A}{\sin A}$

सिद्ध: $\operatorname{cosec} A = \frac{1}{\sin A}$

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उदाहरण (12)

Solveluction

$\because \cos^2 A + \sin^2 A = 1$

$\Rightarrow \cos^2 A = 1 - \sin^2 A$

$\therefore \cos A = \sqrt{1 - \sin^2 A}$

अब, $\because \tan A = \frac{\sin A}{\cos A}$

$\therefore \tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$

फिर, $\because \sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}$

फिर, $\because \sin^2 A + \cos^2 A = 1$

$\Rightarrow \sin^2 A = 1 - \cos^2 \theta$

$\therefore \sin A = \sqrt{1 - \cos^2 \theta}$

उदाहरण (13)

Solveluction

Prove that

$\sec A(1 - \sin A)(\sec A + \tan A) = 1$

L.H.S $= \sec A(1 - \sin A)(\sec A + \tan A)$

$= \frac{1}{\cos A}\left(1 - \sin A\right)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \frac{1 - \sin A}{\cos A} \times \frac{1 + \sin A}{\cos A}$

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$$= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A} = \frac{(1)^2-(\sin A)^2}{\cos^2 A} = \frac{1-\sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A \, 1}{\cos^2 A} = 1 \quad \text{R.H.S proved}$$

Question (15) Solution

$$\text{L.H.S} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} = \frac{(\tan\theta + \sec\theta) - 1 \,(\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta) + 1 \,(\tan\theta - \sec\theta)}$$

$$= \boxed{\frac{\tan\theta + \sec\theta - \tan\theta + \sec\theta}{\tan\theta - \sec\theta + \tan\theta - \sec\theta}}$$

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$$\frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta} = \frac{}{\sec\theta - \tan\theta}$$

Ex - 8.4

① त्रिकोणमितिय अनुपातों में $\sin A$, $\sec A$ और $\tan A$, $\cot A$ के पदों में व्यक्त कीजिए

Solve

formula

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$\Rightarrow \sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1 + \cot^2 A}}$$

और, $\tan A = \frac{\sin A}{\cos A} =$

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उदाहरण: (12)

$$\cos A = \sqrt{1 - \sin^2 A} \quad \text{Ans}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$= \frac{\sin A}{\sqrt{1^2 - \sin^2 A}}$$

$$\sec = \frac{1}{\cos A}$$

$$= \frac{1}{\sqrt{1 - \sin^2 A}}$$

उदाहरण: (13)

$$L.H.S = \sec A(1 - \sin A)(\sec A + \tan A)$$

$$= \frac{1}{\cos A}\left(1 - \sin^2 A\right)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$= \frac{1}{\cos A}(1 - \sin^2 A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1 + \sin A}{\cos A}\right)$$

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(14)

$$= \frac{1 - \sin^2 A}{\cos^2 A}$$

$$= \frac{\cancel{\cos^2 A}}{\cancel{\cos^2 A}} = 1$$ R.H.S proved

अथवा - (14)

$$L.H.S = \frac{\cot A - \cos A}{\cot A + \cos A}$$

$$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

$$= \frac{\cos A \left(\frac{1}{\sin A} - 1\right)}{\cos A \left(\frac{1}{\sin A} + 1\right)}$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$$= R.H.S \text{ proved}$$

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उदाहरण (15)

$$\text{L.H.S} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

$$= \frac{\frac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{\sin\theta + \cos\theta - 1}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\sec\theta + \tan\theta) - 1}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{1 - \sec\theta + \tan\theta}$$

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L.H.S

$$= \frac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - \sec\theta + \tan\theta]}{1 - \sec\theta + \tan\theta}$$

$$= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta}$$

R.H.S proved

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प्रश्नावली - 8.4

(2) $\angle A$ के अन्य सभी त्रिकोणमितिय अनुपातों को Sec A के पदों में लिखिए

Solveution

सूत्र, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ की सहायता से

$\Rightarrow \sin A = \cos A \times \tan A = \frac{1}{\sec A} \times \tan A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$ Ans

$\cos A = \frac{1}{\sec A} = \frac{1}{\sec A}$ Ans

$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}{\cos A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \frac{1}{\cos A}$

$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \frac{1}{\cos A} = \sqrt{\sec^2 A - 1}$ Ans

$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec}}{\frac{\sqrt{\sec^2 A - 1}}{\sec}} = \frac{1}{\sqrt{\sec^2 A - 1}}$

$\text{Cosec } A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec}} = \frac{\sec}{\sqrt{\sec^2 A - 1}}$

(3) मान निकालिए :

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 27^\circ + \cos^2 73^\circ}$

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$$= \frac{\sin^2 63^\circ + \sin^2(90^\circ - 63^\circ)}{\cos^2 27^\circ + \cos^2(90^\circ - 27^\circ)}$$

$$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\cos^2 27^\circ + \sin^2 27^\circ}$$

$$= \frac{1}{1} = 1$$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin 25^\circ \cdot \cos 65^\circ + \cos 25^\circ \cdot \sin 65^\circ$

$= \sin 25^\circ \cdot \cos(90^\circ - 25^\circ) + \cos 25^\circ \cdot \sin(90^\circ - 25^\circ)$

$= \sin 25^\circ \cdot \sin 25^\circ + \cos 25^\circ \cdot \cos 25^\circ$

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1$

(4) (i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(1) Ans: → (B)

Sol^n: $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$

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(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)=$

(A) 0 (B) 1 (C) 2 (D) -1

(ii) Ans: → (C)

Sol^n: - $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec})$

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\Big/\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin+\cos\theta-1}{\sin\theta}\right)$$

$$= \frac{[(\cos\theta+\sin\theta)+1]}{\cos\theta}\left[\frac{(\cos\theta+\sin\theta)-1}{\sin\theta}\right]$$

$$= \frac{(\cos\theta+\sin\theta)^2-1^2}{\cos\theta\cdot\sin\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta+2\cdot\cos\theta\cdot\sin\theta-1}{\cos\theta\cdot\sin\theta}$$

$$= \frac{1+2\cos\theta\sin\theta-1}{\cos\theta\sin\theta}$$

$$= \frac{2\cos\theta\cdot\sin\theta}{\cos\theta\cdot\sin\theta} = 2 \quad \text{R.H.S Proved}$$

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(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) $\sec A$ (B) $\sin A$ (C) $\operatorname{cosec} A$ (D) $\cos A$

(iii) Ans: → (D)

Sol^n: — $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

(iv) Ans: —— (D)

Sol^n: —

$$\frac{1 + \tan^2 A}{1 + \cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

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(5) (i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

L·H·S $= (\operatorname{cosec}\theta - \cot)^2$

$= \left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{1+\cos^2\theta}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$

R·H·S Proved

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

L·H·S $= \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$= \dfrac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

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$$= \frac{\cos^2 A + 1^2 + 2.1.\sin A + \sin^2 A}{(1+\sin A)\cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2\sec$$

R.H.S proved

$$\frac{2}{\cos A} = 2 \times \frac{1}{\cos A}$$

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(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

L.H.S $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{-\sin\theta(\sin\theta-\cos\theta)}$ $\quad \cos\theta-\sin\theta = -\sin\theta+\cos\theta = -(\sin\theta-\cos\theta)$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta}\right]$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)\{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\}}{\cos\theta\sin\theta}\right]$

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(iii)

$$= \frac{1 + \sin\theta\cos\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\cos\theta \sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} + 1$$

$$= \sec\theta \cdot \operatorname{cosec}\theta + 1$$

$$= 1 + \sec\theta \operatorname{cosec}\theta$$

$$= \text{R.H.S proved}$$

(iv)

$$\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$$

$$\text{L.H.S} = \frac{1+\sec A}{\sec A}$$

$$= \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{(1+\cos A)(1-\cos A)}{1 \cdot (1-\cos A)}$$

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$$= \frac{1-\cos^2 A}{1-\cos A}$$

$$= \frac{\sin^2 A}{1-\cos A}$$

$$= \text{R.H.S proved}$$

(v) $$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

L.H.S $$= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\dfrac{\cos A - \sin A + 1}{\sin A}}{\dfrac{\cos A + \sin A - 1}{\sin A}}$$

$$= \frac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - 1}{(1 - \operatorname{cosec} A + \cot A)}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 - \operatorname{cosec} A + \cot A)}$$

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$$= \frac{(\operatorname{cosec}A + \cot A) - (\operatorname{cosec}A + \cot A)(\operatorname{cosec}A - \cot A)}{(1 - \operatorname{cosec}A + \cot A)}$$

$$= \frac{(\operatorname{cosec}A + \cot A)\left[1 - (\operatorname{cosec}A - \cot A)\right]}{(1 - \operatorname{cosec}A + \cot A)}$$

$$= \frac{(\operatorname{cosec}A + \cot A)\cancel{\left[1 - \operatorname{cosec}A + \cot A\right]}}{\cancel{(1 - \operatorname{cosec}A + \cot A)}}$$

$$= \operatorname{cosec}A + \cot A$$

$$= \text{R.H.S proved}$$

(vi) $$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

$$\text{L.H.S} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{(1)^2 - \sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \sqrt{\left(\frac{1+\sin A}{\cos A}\right)^2}$$

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$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

$$= \text{R.H.S proved}$$

(vii) $$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

$$\text{L.H.S} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,[2(1-\sin^2\theta) - 1]}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,[2 - 2\sin^2\theta - 1]}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(1 - 2\sin^2\theta)}$$

$$= \tan\theta$$

$$= \text{R.H.S proved}$$

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(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A$

$= \sin^2 A + \cos^2 A + \sec^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + 2 \cos A$

$= 1 + 1 + \tan^2 A + 1 + \cot^2 A + 2 \times 1 + 2 \times 1$

$= 1 + 1 + \tan^2 A + 1 + \cot^2 A + 2 + 2.$

$= 7 + \tan^2 A + \cot^2 A$

$=$ R.H.S proved

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

L.H.S $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$= \left(\dfrac{1 - \sin^2 A}{\sin A}\right)\left(\dfrac{1 - \cos^2 A}{\cos A}\right)$

$= \dfrac{\cos^2 A}{\sin A} \cdot \dfrac{\sin^2 A}{\cos A}$

$= \cos A \sin A$

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$$R.H.S = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}}$$

$$= \frac{1}{\frac{1}{\cos A \sin A}}$$

$$= \cos A \sin A$$

$$= L.H.S$$

$\therefore$ L.H.S = R.H.S proved

$$\frac{1}{\frac{\cos A \sin A}{\cos A \sin A}}$$

$$\frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}}$$

$$\frac{1}{\frac{\sin A \sin A}{\cos A \sin A} + \frac{\cos A \cos A}{\cos A \sin A}}$$

$$= \frac{1}{\tan A + \cot A}$$

(X) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

L.H.S R.H.S M.H.S

$$L.H.S = \frac{1+\tan^2 A}{1+\cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

= R.H.S proved

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$$M.H.S = \left(\frac{1-\tan A}{1-\cot A}\right)^2$$

$$= \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\frac{\cos A-\sin A}{\cos A}}{\frac{\sin A-\cos A}{\sin A}}\right)^2$$

$$= \left[\frac{(\cos A-\sin A)\sin A}{(\sin A-\cos A)\cos A}\right]^2$$

$$= \left[\frac{-(\sin A-\cos A)\sin A}{(\sin A-\cos A)\cos A}\right]^2$$

$$= \left[-\tan A\right]^2$$

$$= \tan^2 A$$

$$= R.H.S \text{ proved}$$

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